

Category theoretic ontology for representation of assessment scales and consensus guidelines in elderly care

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1 Introduction

Kazem Sadegh-Zadeh's approach¹ to *medical language* is a first true and complete attempt to provide concrete and detail insight into clinical practice and medical decision-making based on practiced morality and normative ethics. His 'language of medicine' is combined with 'medical praxiology' in a very subtle way not seen before in these contexts. Our discussion on ontology in this paper can be seen, in the small, to relate to epistemological aspects in Sadegh-Zadeh's presentation on epistemology, and his book at large will be a continuous inspiration for our further efforts in this area. Our discussion is not broadly in medicine, and, in fact, not within medicine only. Ageing and elderly care is a typical area where 'medical' and 'social' needs to join forces, and we will point out hopefully some interesting aspects in this borderline.

The reason for this paper is at least two-fold. On the one hand, we build upon the extension of general logic, into the so called generalized general logic², and commit ourselves to be extremely aware of the roles as represented by metalanguage for logic and object language actually describing all building blocks of logic. On the other hand, we show how such a pure category theoretic approach to ontology provided by this generalized general logic can be used for uncertainty based information and knowledge representation and, accordingly, how it is used in decision-making in health and social care. Our examples are drawn from management of assessment scales and consensus guidelines in care of older people. In doing this, we then also point out the informal logic character of international standards of medical ontology, and explain e.g. why logical modelling of uncertainty becomes too ambiguous in such informal frameworks.

Computerized decision-making in social and health care is traditionally viewed with ontology not as part of underlying logics for decision-making, but rather as standards and terminologies including skeletons and frameworks of informal relational information structures. We bring these views to the point where data and information are seen residing in the underlying signatures of logic, thus providing a strict basis for producing terms and sentences, in turn appearing in reasoning empowered by a selected proof calculus.

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² [2]

Perhaps the most important point here is that there is not a single logic to model all information and knowledge, and respective professional and ontology user cannot be expected to adapt to one such universally chosen logic. This is not a question about a professional not being able to adapt to such a chosen logic, but rather supports the view that context of decision-making is not related to semantics of data, but to appropriate and necessary selections of underlying logics of reasoning. The realm of general logic also and in particular in its generalized form makes explicit use of mappings from one logic to another, so that the status of mutual understanding between professional, possibly using different logics, resides in those morphisms between respective logics.

The paper outline is as follows. We first recall some history of logic and developments of formal logic, and how we arrive at using our framework for generalized general logic. In doing so, we then also provide the overall scope of that formal logic framework. We then provide some related views on existing medical ontology in order to show how these ontologies are logically very restricted and informal. This is followed by a section on ageing, where we provide necessary background to health and social care of older people, and also provide some detailed information e.g. on assessment scales used in these *observe-assess-decide* processes in a care of older people. The following section then goes into the strictly formal aspects of generalized general logic, and we will see how assessment scale based information, knowledge representation and reasoning can be managed in this purely categorical framework. This also reveals where and how modelling of uncertainty can be formally incorporated into this machinery.

2 Logic defines ontology

Programming in logic is manipulation of terms, and substitution with terms. Classical terms won't suffice. An ontology building upon classical terms, trying to enhance missing parts in the underlying structures by being clever about inference, becomes logically sterile and basically useless in formal frameworks. We also need to make a distinction between imprecise or vague information, and being formal and accurate in reasoning with vague values. Furthermore, a value may be vague as produced by a crisp operation, or a value is vague since the underlying operation is vague.

From formal point of view this is all about underlying categories and monads, and in this paper we will continue investigations³ showing how the signatures reside in term monads over chosen categories. Our approach is thus monadic, and we consider monads over suitable categories.

Ontology can be informal or formal, and our approach is that ontology must be nothing but formal and mathematically unambiguous. To be more precise concerning *logic*, let us describe *what* logic is and *how* logic is defined.

Firstly, there is not a single logic for everything. Secondly, we need to distinguish logic as the basis for mathematical reasoning from logic as dedicated to

³ [3]

theory development and programming of rule bases. In all this, it is important to understand what is the object language for logic, and what is the metalanguage supporting that object language.

The logic for mathematical reasoning goes back to Aristotle and even before those times to pre-Socratic times when e.g. reductio ad absurdum was used by Zeno. Mathematicians, like Szabó⁴, say Aristotle didn't say all that much that influenced modern developments of logic, whereas philosophers, like Hintikka, read lots between the lines and provide far going interpretation about what Aristotle said e.g. about deictics (syntactic, roughly speaking) and apodeictics (semantics, roughly speaking). Our take on logic must be the mathematical one, since the philosophical approach doesn't primarily support formalism and ontology based strict representations. Logic becomes formal logic during late nineteenth century when Frege⁵ defines what we now call *first-order logic*. This logic was originally intended as logic only for mathematical reasoning, i.e. logic for mathematics. At the change of the century, Hilbert pointed out the difficulties concerning natural numbers and logic, and the question was "Which comes first?". The metalanguage for this first-order logic is not existing *per se*, but we rather have a situation where the non-meta based object language for logic is constructed, and leading to formal difficulties and even paradoxes, which are then rendered, and the formal basis for the object language is reiterated to avoid these difficulties. This process of finding difficulties followed by rendering these difficulties continued for decades, and when Hilbert some fifty years later (with Bernays) was finishing work on set theory and foundations of mathematics⁶, the question remained still unanswered. Between Frege's *Begriffsschrift* and Hilbert-Bernays' *Grundlagen*, lots of things happen in the discussion on logic. Peano⁷ did his axioms for natural numbers, Russell entered the debate through paradoxes and many others contributed to these discussions. Some computationally interesting things happen also late at those times, e.g. by Schönfinkel⁸, a frequent visitor to Hilbert in Göttingen, and his work on combinatory logic, later transformed by Curry in his thesis⁹ (supervised by Hilbert and Bernays) providing groundwork for λ -calculus, using only a subset of Schönfinkel's combinators, and thereby type theory was born. Curry together with Howard later showed how propositions can be interpreted as types, an observation that has seduced computer scientist almost a century now.

Logic as dedicated to theory development builds upon a very precise meaning of what logic really is. Formally (and computationally) speaking, logic consists of

- its signature with sorts (types) and operators,
- algebras providing the meaning of the signature,

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⁵ [5]

⁶ [6, 7]

⁷ [8]

⁸ [9]

⁹ [10]

- all terms constructed (syntactically) using operators in the signature, and the resulting algebraic interpretations (semantics) of these terms,
- all sentences having terms as building blocks, and again equipped with the corresponding meaning (algebras) of these sentences,
- all theoremata being conglomerates of sentences as used in reasoning,
- entailment as the relation between theoremata representing what we already know, and sentences representing knowledge we are trying to arrive at,
- satisfaction as the semantic counterpart to entailment providing the notion of valid conclusions,
- axioms saying what we take for granted at start,
- inference rules saying how we can jump to conclusions in a chain of entailments, and these rules being selected so as to ensure equality (i.e. the so called soundness and completeness of the logic) between the entailment and satisfaction relations (equality cannot always be achieved as the completeness part of logic is sometimes difficult to reach).

In a subsequent section we will make all these notions precise using category theory as metalanguage.

At this point, note how the signature and terms are ingredients for information (as "data") in databases and database theory, where further inclusions of sentence and theoremata are ingredients for knowledge ("representation"). Entailment and inference rules are then finally need in order to "compute" of "infer" with knowledge. This means that we must be careful when we speak about "guidelines" since we must reveal whether we speak only about the knowledge representations involved or also about how to deduce new knowledge using these known representations. Unfortunately, most guidelines are only knowledge representative.

For this notions to enable unambiguous formalism, category theory plays a fundamental role as the metalanguage (in turn with the Zermelo-Fraenkel set theory as the metalanguage for category theory, and so on, hierarchically) for logic formalism, as the formal (and computable) notions of *term*, *sentence* and *theoremata* are given by functors extendable to *monads*, and in the case of theoremata even to *partially ordered monads*¹⁰.

An important part of this approach to ontology is also its capacity and capability to embrace modelling of uncertainty and non-determinism.

At this point we should again underline that we do not have a single logic, as dedicated to theory development, covering reasoning within all applications. The situation is very much the opposite, namely, in that site response and site management must be allowed to use different logics, and crisis response and management logic again differs rather significantly from site response and management logics. The important property in these respects is that there are mappings between these logics so that knowledge, represented in a particular logic, can be carried over to be represented in another logic, understood by other users and stakeholders. The categorical and monadic approach to logic is critical in partic-

¹⁰ [11]

ular for these homomorphic transformations as represented enabled by functors and represented mostly by natural transformations between them.

We thus define ontology as information and knowledge encoded using a particular logic.

To summarize our main claim in this section, we see logic for mathematics being the first-order approach developing hand in hand with axiomatic set theory, providing ZFC, Zermelo-Fraenkel's set theory including the Axiom of Choice, as the metalanguage (including appropriate intuitions for *conglomerates* and *universe*) for the object language *category theory*. In turn, when we move over to defining *formal logic*, category theory becomes the metalanguage for the object language *generalized general logic*.

In this strictly hierarchical approach we forbid moving back and forth, as Gödel frequently did, and indeed remain strict when representation terms, sentences and proofs in logic. Gödel's numbering indeed comes to a proof calculus, using proof trees and provides numberings for sentences appearing in proof trees, then producing predicates involving these numberings, and goes back to the set of sentences and throws this new sentences into the bag of old sentences. This was allowed one hundred years ago, and some still allow it. We don't, and indeed for our families of logic enabled by the framework of generalized general logic. Whatever happened before ZFC became ZFC, is here not of our concern. We trust ZFC and we trust ZFC as the metalanguage, not *a* metalanguage, for category theory. And generalized general logic must use categorical notions only, and as such based on ZFC. No by-passing of this principle is allowed.

3 Ontology in the medical domain

Obviously, ontology as traditionally known e.g. in the medical domain as built upon standards like HL7, SNOMED CT and openEHR, or OWL and RDF for web ontology, are not fully logical. They are only partially logical in that even the underlying signature for encoding their vocabularies is treated informally with concepts being more like atoms, and sentence constructions as typically represented by relations, like `IS_A` in SNOMED CT, e.g. in statements like `open fracture of foot IS_A fracture of foot IS_A injury of foot IS_A disorder of foot`. The term `open fracture of foot` is then more typically used in first response for decision-making related to pre-hospital interventions for open fractures, whereas `disorder of foot` more levels involving expertise for orthopedics.

OWL and SNOMED CT have adopted variants of description logics for their partial ontologies. The variant EL++ is favored in OWL, and has recently (because of that use within OWL) also been adopted for SNOMED CT. However, OWL is more tightly bound to EL++, whereas SNOMED CT is still only intentionally bound to EL++.

It should also be noted that description logic is not a formal logic as described above. Description logic doesn't even have a formal involvement of signature as they use *concept* as a primitive notion. Concepts are used as terms and sentences

are relational only, which means description logics appear (intentionally) as kind of an informal subset of first-order logic. Description logic further does not really recognize the distinction between logic as the basis for mathematical reasoning from logic as dedicated to theory development and programming of rule bases. It is surely intended for the latter, but it is constructed in the manner like the former. Even worse, description logic has severe difficulties to include reasoning mechanisms, which means that this logic as a partial ontology remains on the logic levels including signatures, terms and sentences only, and even being rather informal about them.

Partial ontology is taken to mean information (databases) and knowledge (guidelines) encoded in logic where parts of the logic structure are missing. The traditional meaning of ontology, such as in web ontology, indeed either completely neglects or is intentionally informal in particular about the sentential and inferential parts of logic. Partial ontology, in particular as seen in the case of SNOMED CT, is more like a mereology since the meronomic type hierarchies in SNOMED CT still seek to find a proper inclusion of deductive elements, and therefore in some sense disqualifies to be called ontology. Another way of speaking is to say that nomenclature is not sufficient, since we need a *calculus of nomenclature*¹¹.

4 Ageing

The overall objectives of the Observe-Assess-Decide (OAD) process in elderly care is to provide a complete system for observation, assessment and decision-making focused on home care and prolongation of independent living, by providing a necessary and sufficient ontology and assessment scale based information, thereby enabling well-founded predictions and continuous monitoring of decline and progrediation on both individual as well as group level. OAD aims to facilitate both dynamical settings of individual care level for care provision at point-of-need as well as demographic change based accurate socio-economic modelling supporting strategic regional management of ageing.

The lack of regional strategies together with scattered and unstructured guidelines for prevention, detection and intervention related to older persons' decline in cognitive and functional capabilities, are the most serious obstacles in the way of a sustainable development of supportive environments for the elderly. Further, the lack of well-structured guidelines and well-organized utility of assessment and, in particular, rigorous assessment based decision-making and care provisioning, leads to overlaps and inefficiency, and even worse, to subjective decision-making and care processes that cannot be measured nor evaluated.

There are assessment scales that are more suitable for home care, where other scales might be seen more suitable for nursing homes, and so on and so forth. For example, on non-cognitive aspects of dementia, some of the first parts of NPI

¹¹ Stanislaw Lesniewski (1886-1939), a Polish logician contemporary with Alfred Tarski and Jan Lukasiewicz, used ontology in the sense of a *calculus of names* in his Grundzüge [12].

might be more effectively used in home care, whereas Behave-AD and CMAI (focusing on agitation) are more useful in residential care and hospital wards.

Optimal use of OAD's gerontechnological platform¹² relies on the specific competences as represented by respective professionals and professional groups. Elderly care includes personnel of various fields, skills and expertise e.g. social workers, nurses, gerontologists, therapists, psychologists and physicians, general practitioners, neurologists and geriatricians. It should be noted that the home care staff in its vast majority consists of a selected mix of social workers and nurses, and thus social care becomes comparably important together with health care. Also in residential and nursing homes, social and health care should be in balance, while in hospital wards the provision of health/medical care is usually seen more important.

In comparison with the working population, older people are more likely to suffer from a wider range of diseases. Public diseases, including problems caused by, and related to metabolic syndromes, diabetes, obesity, malnutrition and sleep deprivation, usually appear accompanied with cardiovascular diseases (cardiac failure, atherosclerosis, vascular disease and hypertension). Ageing then comes more and more with cerebrovascular disease, COPD (Chronic Obstructive Pulmonary Disease), and various frailty syndromes including osteoporosis and sarcopenia, with the risk of a potential fall to be on the increase, having severe effects on the care levels. For instance after a fall, the need for physical exercise and rehabilitation increases. Furthermore, various forms of cancer appear more frequently, and palliative treatment in the last stages of cancer is one of the main reasons why a transfer to domestic environment is often preferred by the patient.

While monitoring of health condition and follow-up of interventions can be supported by devices and ubiquity (e.g. glucose meters for diabetes and monitoring sleep disorders by using sensors implanted in beds), the detecting and monitoring of cognitive decline and psycho-geriatric diseases require assessment scales.

Cognitive decline in MCI (Mild Cognitive Impairment) stages and as appearing in different severity degrees in Alzheimer's disease and other dementia types are typical for old age, and are further accompanied by psycho-geriatric problems such as depression, delirium, and various non-cognitive¹³ symptoms.

¹² <http://www.fourcomp.com/oad/>

¹³ We prefer the neutral and widely accepted term "non-cognitive symptoms of dementia", even if the concept of Behavioral and Psychological Symptoms in Dementia (BPSD) has been defined e.g. by the International Psychogeriatric Association (IPA). BPSD was intended to cover a heterogeneous range of psychological reactions, psychiatric symptoms, and behavior occurring in people with dementia of any aetiology. However, BPSD has become controversial as it invites to treating a syndrome or disorder, thus neglecting distinctions between individual symptoms. Aetiological homogeneity in these respects is now seen to be rather unlikely. Research and trials on BPSD might even lack external validity since pharmacological trials showing the effectiveness of psychotropic drugs in treating these symptoms have been based on

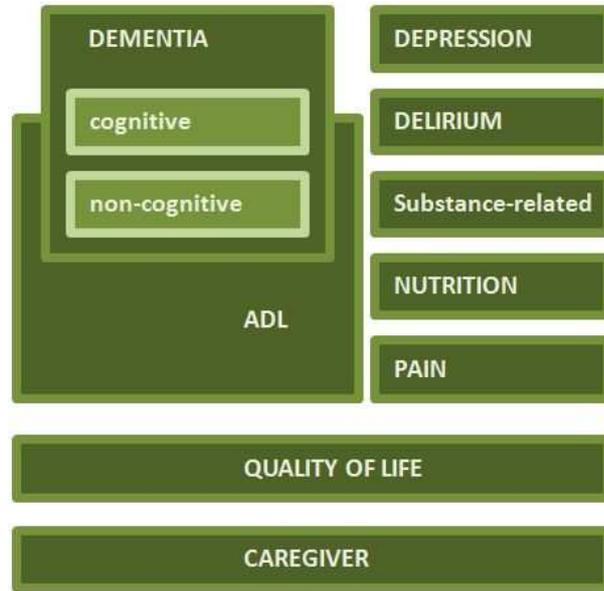


Fig. 1. Framework for assessment scales.

Figure 1 illustrates the minimal set of assessment scales, which usually comprises of some ADL (Activities of Daily Living) scales combined with suitable cognitive scales like MMSE¹⁴. Combination scales, like the CDR¹⁵ (Clinical Dementia Rating) for ADL/DEMENTIA, are also widely used. Non-cognitive signs are captured e.g. by NPI¹⁶ (Neuropsychiatric Inventory), CMAI¹⁷ (Cohen-Mansfield Agitation Inventory) and BEHAVE-AD¹⁸. NPI is particularly useful in home care¹⁹. Depression is usually captured in its own right as a non-cognitive aspect of dementia, where e.g. GDS²⁰ (Geriatric Depression Scale) is widely used in home care. Depression is known to accelerate cognitive decline. Nutrition scales are important, as are the scales for social conditions, and so on and so forth.

The selection of assessment scales to be used is of utmost importance and must be optimized with respect to professional resources available in the partic-

summations representing the variety of BPSD symptoms rather than using particular scales. Notably, BPSD is not included as a term in medical databases.

¹⁴ [13]

¹⁵ [14]

¹⁶ [15]

¹⁷ [16]

¹⁸ [17]

¹⁹ [18]

²⁰ [19]

ular service field where the OAD gerontechnological platform is to be installed and used.

Accurate monitoring of assessment scale based data supports also dementia differential diagnosis²¹ according to guidelines as provided e.g. by DSM-IV²² and NINCDS-ARDRA. Early detection of dementia is important e.g. to achieve favorable effects of pharmacologic treatment by cholinesterase inhibitors (for Alzheimer’s disease and Lewy body dementia).

5 Generalized general logic

A signature $\Sigma = (S, \Omega)$ consists of sorts (types) in S and operators in Ω . Here S is a *set* in the sense of ZFC. On the other hand, Ω is not just a set, but in fact an object in an underlying category. If this category is **Set**, the ordinary category of sets and functions, in the sense of ZFC, then Ω is a set just like S also is just a set.

Now let **answer** $\in S$ be a sort, and let **no**, **yes** $:\rightarrow$ **answer** be two constant (0-ary operators) of sort **answer**. In GDS, the first question is ”*Are you basically satisfied with your life?*”. The observation about the target older person may be *Yes*. This gives no room for representing uncertainty about the observation.

For each sort **s** $\in S$, the algebra \mathfrak{A} provides the sort with a domain $\mathfrak{A}(\mathbf{s})$, which typically is seen as a set, i.e. an object in **Set**. Operators $\omega : \mathbf{s}_1 \times \cdots \times \mathbf{s}_n \rightarrow \mathbf{s}$ are then provided with a meaning $\mathfrak{A}(\omega) : \mathfrak{A}(\mathbf{s}_1) \times \cdots \times \mathfrak{A}(\mathbf{s}_n) \rightarrow \mathfrak{A}(\mathbf{s})$, i.e. a morphism in **Set**. Again, there is no a priori reason why **Set** must be fixed as the underlying category for algebras of signatures.

In order to see the difference in using other underlying categories than **Set**, let us first look at the term functor $T_\Sigma : \mathbf{Set} \rightarrow \mathbf{Set}$. The term functor can be constructed in a strict categorical fashion²³, so that $T_\Sigma X$ becomes the set of all terms over the set X of variables, i.e. X being an object of **Set**.

To continue the example above, the term **yes** is recorded as the observation. Let now the underlying category be changed to **Set**(L), the Goguen category, where L is a suitable lattice. Objects in **Set**(L) are pairs (A, α) where $\alpha : A \rightarrow L$ is a mapping. Morphisms $f : (A_1, \alpha_1) \rightarrow (A_2, \alpha_2)$ are mappings $f : A_1 \rightarrow A_2$ such that $\alpha_2(f(a)) \geq \alpha_1(a)$ for all $a \in A_1$. Note that **Set** is not isomorphic to **Set**($\{0, 1\}$). Assume for instance that $L = \{\mathbf{absent}, \mathbf{possible}, \mathbf{probable}, \mathbf{present}\}$, with the names for the elements in L really being just names or symbols for points in L . The set S of sorts remains a set, but the ’set’ of operators becomes an object of **Set**(L), so we now have (Ω, ϑ) , for some $\vartheta : \Omega \rightarrow L$, as the operator object in **Set**(L). The constant **no** $:\rightarrow$ **answer** is now recorded as $\vartheta(\mathbf{no}) \in L$. Even more so, ϑ should now be seen as *specific for an observer*. There may indeed be (at least) two observers, **Flo** and **Rence**, so that $\vartheta_{\mathbf{Flo}}$ and $\vartheta_{\mathbf{Rence}}$ bind uncertainty values of **yes** to the specific observer. Thus, we may have $\vartheta_{\mathbf{Flo}} = \mathbf{present}$ and

²¹ [20]

²² [21]

²³ [11]

$\vartheta_{\text{Rence}} = \text{absent}$. Flo, an experienced home carer may have recorded the observation after having cared for the patient over the past five years, whereas Rence a primary care physician may have seen the patient for the first time in relation to updating a prescription for hypertension.

The term functor is now $\mathsf{T}_\Sigma : \mathbf{Set}(L) \rightarrow \mathbf{Set}(L)$, and in fact, these functors can be extended to monads, and we speak of the term monad over $\mathbf{Set}(L)$.

At this point we are able to see alternatives for incorporating models of uncertainty. An operator $\omega : \mathbf{s}_1 \times \cdots \times \mathbf{s}_n \rightarrow \mathbf{s}$ residing in (Ω, ϑ) is *fully*²⁴ fuzzy, and $\vartheta(\omega)$ represents the uncertainty of that particular operator. We can then speak about *fuzzy operators*. If our underlying signature is the signature for natural numbers, i.e. $S = \{\text{nat}\}$ and $\Omega = \{0 : \rightarrow \text{nat}, \text{succ} : \text{nat} \rightarrow \text{nat}\}$, then $\vartheta(0)$ and $\vartheta(\text{succ})$ are equipped with uncertainties, and the uncertainty of a term like $\text{succ}(\text{succ}(\text{succ}(0)))$ can be computed from the basic uncertainties of the operators. Note here also that a 'set' Y of variables is an object of $\mathbf{Set}(L)$, so $Y = (X, \beta)$ with $\beta : X \rightarrow L$. This means not only that a variable is uncertain but variable substitutions, and computing as based on variable substitutions become uncertain. This view of a basis for *fuzzy arithmetic* is entirely different from traditional set-theoretic notions of uncertainty.

An alternative way to incorporate modelling of uncertainty as compared with *uncertain computation* is to enable *computing with uncertainties*. This is essentially done with composing suitable monads with the term monad over \mathbf{Set} . Indeed let $\mathsf{T}_\Sigma : \mathbf{Set} \rightarrow \mathbf{Set}$ be the term monad over \mathbf{Set} , as used above, and let $\phi : \mathbf{Set} \rightarrow \mathbf{Set}$ be another monad over \mathbf{Set} that is *composable*²⁵ with the term monad in the sense of the composed functor $\phi \circ \mathsf{T}_\Sigma : \mathbf{Set} \rightarrow \mathbf{Set}$ being extendable to a monad. In case of *fuzzy*, ϕ is typically selected to be the fuzzy powerset functor L . Now a variable set X is an object of \mathbf{Set} , as just an ordinary set, but substitution is e.g. a morphism $\sigma : X \rightarrow \mathsf{LT}_\Sigma X$ that maps $x \in X$ into $\sigma(x) = \{0/0.7, \text{succ}(0)/0.5, \text{succ}(\text{succ}(0))/0.2\}$, using Zadeh's original notation for fuzzy sets and assuming $L = [0, 1]$, the unit interval. In this substitution we have x is bound to '0' with uncertainty value 0.7, x is bound to '1' with uncertainty value 0.5 and to '2' with uncertainty value 0.2. Note how this then eventually leads to *arithmetic with fuzzy*, entirely distinct conceptually from fuzzy arithmetic. In the case of fuzzy arithmetic it is further far from clear that we can do with the nat sort only, or if we need some additional fuznat sort, or even some form of type constructor $\text{fuz} : \text{type} \rightarrow \text{type}$ on a second level of signatures, where nat would be integrated as a constant operator $\text{nat} : \rightarrow \text{type}$, and we obtain a new type $\text{fuz}(\text{nat})$ for which the algebra then is something like $\mathfrak{A}(\text{fuz}(\text{nat})) = \mathfrak{A}(\text{fuz})(\mathfrak{A}(\text{nat}))$. If $\mathfrak{A}(\text{fuz}) = L$ and $\mathfrak{A}(\text{nat}) = N$, then clearly $\mathfrak{A}(\text{fuz}(\text{nat})) = LN$, which shows that the meaning of algebra also becomes extended when going in these generalizing directions. This can in fact be formalized, as is done in forthcoming papers. The distinction between *computing with uncertainty* and *uncertain computation* is indeed a first step towards iden-

²⁴ According to Lawrence Neff Stout's vocabulary.

²⁵ Composability of monads is subject to certain conditions, so called *distributive laws*, first studied by Jon Beck [22].

tifying various paradigms for substitutions²⁶ that are underlying for the whole machinery of generalized general logic.

For assessment scales, like e.g. the GDS scale for depression with a total of 30 questions, or one of its subsets GDS-15, GDS-10, or even GDS-4 with just 4 questions. At least two 'positive' answers out of 4 "raises a flag". This is not saying depression is there, and clearly this is not a step in mood diagnostics of depression e.g. according to the DSM-IV guidelines. It is saying "pay attention" since we have something here that falls under the umbrella of depression, and we know that depression accelerates memory loss.

We will not formally define sentences, but here only draw the attention to sentences being defined by a sentence functor Sen with domain being the category of monads over a fixed underlying category. The typical example is $\text{Sen}(\mathbb{T}_{\Sigma}) = \text{id}^2 \circ \mathbb{T}_{\Sigma}$ in the case of producing sentence for equational logic. First-order logic and various extensions can be defined in the context of generalized general logic, and this first-order logic must not be confused with the first-order logic appearing together with ZFC. Note also that natural numbers can be made to "reappear" e.g. in equational logic as defined above, and this Peano arithmetics is then not to be confused with Peano's arithmetic as appearing in the realm of ZFC, for which Gödel's self-referential numbering approaches are accepted.

At this point we can proceed to be extremely formal, but this requires writing space far beyond what is available for this paper. We may for the purpose of this paper, a bit informally, picture sentences like $\text{GDS-4}(\text{number_of_positive_answers}) \geq 2$, and then we intuitively see how the 'truth' of this sentence is related to our observations.

Suppose we are faced with a dementia case, and we want to differentiate between Alzheimer's Disease (AD) and a Vascular Dementia (VaD). Inhibiting drugs may be used for AD, but they are not suitable for VaD. Also the progression of AD differs from that of VaD, so a long-term treatment plan for AD may differ from a corresponding plan for VaD, since e.g. behavioral syndromes related to a VaD patient may be more clear than for a AD patient. We may then have sentences formulated based on observations of depression (e.g. by a GDS scale), hypertension, information about a previous stroke, and memory loss (e.g. by the MMSE scale). Depending on how many of these sentences tend to show 'truth', we will make a basic judgment about VaD and AD, which may be very useful e.g. for home care decision-making before possible neurological statements are at hand.

Concerning entailment, and even more informally speaking, we then have conclusions like

$$\{\text{depression, stroke, hypertension, cognitive_failure}\} \vdash \text{VaD}$$

Now the premises, called *theoremata*, fit into a logic shared by a particular professional. Back to F_{10} and Rence , we can then imagine the respective conclusions

$$\{\text{depression}_{\text{F}_{10}}, \text{stroke}_{\text{F}_{10}}, \text{hypertension}_{\text{F}_{10}}, \text{cognitive_failure}_{\text{F}_{10}}\} \vdash_{\text{F}_{10}} \text{VaD}_{\text{F}_{10}}$$

²⁶ [23]

$\{\text{depression}_{\text{Rence}}, \text{stroke}_{\text{Rence}}, \text{hypertension}_{\text{Rence}}, \text{cognitive failure}_{\text{Rence}}\} \vdash_{\text{Rence}} \text{VaD}_{\text{Rence}}$

where $\text{depression}_{\text{Flo}}$ and $\text{depression}_{\text{Rence}}$ embrace observations $\varsigma_{\text{Flo}}(\text{no}) = \text{present}$ and $\varsigma_{\text{Rence}}(\text{no}) = \text{absent}$ for the first question in GDS-4. Important here is that the entailment \vdash_{Flo} resides in the logic adopted by **Flo**, and \vdash_{Rence} resides in the logic adopted by **Rence**. The question of which conclusion is 'correct' is rather irrelevant. The interesting aspect is whether or not there is a mapping $\vdash_{\text{Flo}} \rightarrow \vdash_{\text{Rence}}$, or in general a morphism between the logics adopted, respectively, by **Flo** and **Rence**.

The theoremata $\Gamma = \{\text{'depression'}, \text{'stroke'}, \text{'hypertension'}, \text{'cognitive failure'}\}$ has a very special form as it is a subset of $\text{Sen}(\mathbb{T}_{\Sigma})X$, where X is a variable 'set', usually many-sorted. For P denoting the powerset functor, which is extendable to a monad, we have $\Gamma \in \text{PSen}(\mathbb{T}_{\Sigma})X$. There is no reason why theoremata couldn't be given by significantly more complicated monads Φ than just P .

Generalized general logic is $\text{LOGIC} = (\text{Sign}, \text{Sen}, \text{Mod}, \Phi, L, \vdash, \models, \text{ProofCalc})$, where **Sign** is the category of signature, **Sen** is the sentence functor, **Mod** is the functor capturing the generalized notion of corresponding algebras, Φ is the theoremata monad, L is a lattice of external truth values, not to be confused with a possible lattice K appearing in the Goguen category $\text{Set}(K)$. We may have $K = L$, but it is not necessary. The entailment relation is \vdash and the satisfaction relation is \models . **ProofCalc** represents functors and natural transformations adding up to a generalized proof calculus. Details are and must be omitted in this paper, but part of this framework was published by Eklund and Helgesson in 2010²⁷, and further detail are under preparation.

We may construct the category of generalized general logics with corresponding morphisms, so that e.g. $\Xi : \text{LOGIC}_{\text{Flo}} \rightarrow \text{LOGIC}_{\text{Rence}}$ captures some kind of *understanding* between **Flo** and **Rence**. The interesting thing here is that $\text{LOGIC}_{\text{Flo}}$ is obviously 'owned' by **Flo** and $\text{LOGIC}_{\text{Rence}}$ by **Rence**, but who owns the morphism Ξ ? It may perhaps be seen as a convergent dialectics between **Flo** and **Rence**.

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